

Simulation of Oil Production with Simultaneous Inclusion of a Hydrocarbon Constituent Driven Random Environmental Perturbation Value of 0.08

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Abstract

A numerical simulation of two non-linear first order differential equations that described crude oil production with simultaneous inclusion of a hydrocarbon constituent driven random environmental perturbation value of 0.08 was investigated. The study adopted a twine MATLAB solution program to simulate the effect of varying a model parameter value on the quantity of crude oil that is produced in the two connecting crude oil wells. The inclusion of a hydrocarbon constituent driven random environmental perturbation value of 0.08 on the growth rate of the quantity of crude oil produced into the two interacting crude oil wells indicate a dominantly associated increase in the quantity of crude oil that is produced in the unit of millions of barrels as the length of production in the unit of days changes for every thirty (30) days scenario. This result implies that a strong or higher hydrocarbon constituent could lead to a relatively bigger volume of crude oil that is produced for the purpose of enhanced and better revenue derivation from the oil sector. Agencies of the Government that is responsible for the management of oil production in Nigeria should therefore adopt measures that will improve the quality of the hydrocarbon constituent for the purpose of sustainability of the nation's treasury for national growth and development.

Key words; Numerical Simulation, crude oil Production, Hydrocarbon Constituent, Random Environmental Perturbation

1.1 Introduction

The mathematical idea employed by experts in numerical predictions involving the modeling processes can also be linked to the adoption of ordinary differential equation of order 45 (ODE 45) Numerical Simulation to predict the quantity of crude oil that will be produced in the context of two neighbouring crude oil wells within the Niger Delta region of Nigeria. The study of oil production optimization using a mathematical model to analyze crude oil extraction from oil wells under the assumption that the pipe line used suffers from damages of various sizes thus resulting in leakages. Cost associated with the leakages is substantial and renders crude oil extraction cost very large. The total cost per unit time and sensitivity analysis was carried to search for critical factors for optimum extraction. The empirical data suggested that leakage area

and holding cost contribute maximum to the cost and the cost of crude oil render production cost[Shah and Mishra (2012)]

A mathematical model have been developed to characterize the oil production in the Federal Republic of Nigeria, calculated by initiating the dynamic of oil production in million barrels revenue plan cost of oil production in million from 1974 – 1982 in the contest of the Federal Republic of Nigeria. The transport network from origin/destination were taking into account, simulation runs, optimization where also considered in the study Paul *et al* (2018), The study shows that the oil production in million barrels from 1974 – 1982 remained fluctuated as well as oil revenue, cost of production and unit cost of production. This indicates that the trend increases and falls. However, the model gives an easy to handle tool for analyzing the policy implications in accordance with the cost and other coefficient applied or adopted in the model.

Quekpo *et al* (2003) presented a solution methodology for the optimization of integrated oil production system at the design and operational levels using simulation models and optimization algorithms. The simulation model represents an integrated oil production system, drainage area, wellhead assembly and surface facilities. The model involves several sub-models such as reservoir, multiphase flowing pines, choke separator and business economics. The key result shows that there is an increase in the present values of the revenues of 127% and 132% with respect to reference values.

Furthermore, Xu *et al* (2018) examined the numerical simulation of oil well production performance considering Pre-Darcy flow using EDFM (embedded discrete facture model) as a numerical simulation tool. In the study the conservation equation were derives for different media, the Darcy law was used, the solution workflow was showed and the verification presented. Also the Pre-Darcy flow was considered in the matrix. The simulation results of the extended model are compared with that of local grid refined (LGR) method. The oil/water production rate shows a big difference when considering a Darch flow and Pre-Darcy flow.

A prediction on Nigeria's oil depletion based on Hubbert's model and need for renewable energy was examined by Ugochukwu and Ogbonnaya (2011). Data for production rate was sourced from 1958 –2008 for the study. The research employed Hubbert's model that based the peaking of a bell-shaped curve that rises rapidly to a peak and declines just as quickly. Matlab tool was employed in the analysis of data findings include there is an imminent decline in Nigeria's oil reserve since peaking could have occurred or just about to occur.

Also, Al-jarri and Startzman (1997) in a study of Analysis of world crude oil production trends reviews historical crude oil production of 67 countries, these countries virtually supply the entire world's crude. The study uses historical production data with a non-linear least squares curve fitting method to determine the parameters of Hubbert's model. The study arbitrarily classified the goodness of fit of the production trends of all countries into three categories, good, fair and poor. The paper attempt to explain the deviations of actual data, political, economic and exploration policy may be important cancer of tese deviations in production trend. The paper ended with a discussion of non-mechanical factors or Hubbert's model. These factors include sedimentary volume or area of petroleum concentration, exploration efficiency, economic incentives political events and technology.

Finol and Farouq (1997) in a study of numerical simulation of oil production with simultaneous ground subsidence developed two-phases, two-dimensional black oil simulator for simulating reservoir production behaviour with simultaneously occurring reservoir formation compaction and ground subsidence of the surface. The flow equations were solved by alternating direction impact procedure and strongly implicit procedure. The model was used for generating the reservoir formation profiles as well as the ground subsidence bowls for a variety of condition. It was found that the subsidence behaviour strongly depends on the depth of burial. Finally the model was used for simulating the reported oil prediction and subsidence history of the Bolivar coast oil fields in the western Venezuela. Fair agreement was obtained between the observed and the predicted behaviour.

Kingsley – Akpara and Iledare (2014) studied the modelling crude oil production outlook: A case study of the oil and gas industry in Nigeria: This study focus on oil production model which tracks oil exploitation as to facilitate good national economic planning and industry strategy. It also develops an empirical model to describe and explain the competing factors underlying oil production patterns. The study also formulated a production model equation with a linear curve fitting method to estimate the Hubbert's model parameters for Nigeria. The model is also use to forecast future production outlook for Nigeria. The result of the model suggest that the rate of production should get to 2.70 MMSTB/D in the year 2010 which will give rise to 65 billion barrels by the year 2050 knowing the current estimated reserves at 37.2 billion barrels.

Nashawi *et al* (2010) also carried out a study on the forecasting of world crude oil production, using multicyclic Hubbert model, which is against the Hubbert single-cycle model. The result after analysis 47 major oil producing countries estimates, estimated the world's ultimate crude oil reserve 2140 BSTB and the remaining recoverable oil by 1161 BSTB. The result also shows the estimated peak for the world production at a rate of 79mmSTB/D. More so, Jinkai *et al* (2001) applied the method of numerical simulation on soil rim impact on underground gas storage in the Dagang oil field (G 393) was taken as an example to study the oil rim impact mechanism. The oil rim impact mechanism on the seepage of underground gas storage operation was studies using numerical simulation method, the result shows that oil rim may prevent seepage of gas resulting in one way gas on rush, thus affecting the smooth operation of underground gas storage.

Zhao *et al* (2019) carried out a numerical simulation study of heavy oil production by using in-situ combustion. Here the in-situ combustion has been numerically simulated based on the Du 66 block. The effect of production parameters on the heavy oil recovery was comprehensively analyzed. Results show that the flooding efficiency is positively correlated with the formation heterogeneity.

Ovalles *et al* (2001) in a study Extra- heavy crude oil down hole upgrading process using hydrogen donors under steam injection conditions found out the presence of the natural formation (catalysts) and methane (natural gas) as necessary to enhance the properties of the upgraded crude oil. Compositional thermal numerical simulations carried out showed a good match between the calculated and experimental API gravities of upgrade crude oil.

Also Ovalles *et al* (2001) carried out physical and numerical down hole upgrading process using hydrogen donors under cyclic steam injection conditions and the result shows that the use of a

hydrogen donor additive (tetralin) in the presence of methane (natural gas) and mineral formation under steam injection conditions led to gravity of treated extra heavy crude oil. While Jiang et al (2010) used a combination of physical modelling and numerical simulation to systematically study the mechanism of oil displacement in low-permeability reservoirs.

Cai et al (2001) carried out wet combustion experiments on heavy oil in the Hekou oil field by using physical simulation technology. The effect of the parameters on the reservoir performance under wet spontaneous combustion was studied. The results shows that wet combustion can recover heat more effectively than dry combustion. It reduces fuel consumption and air consumption, and also improves oil recovery.

Yang *et al* (2016) also carried out a study on the Numerical Simulation and Optimization of enhance oil recovery by the situ generated CO₂ Huff-n-puff process with compound surfactant. The result shows a desirable agreement between the field applications and simulated results.

Ayhan and Hishan (2017) in a study of Optimization of crude oil refinery products to valuable fuel blends comes up with the finding that the optimization of crude oil blends and maximization of low cost refinery intermediaries are the basic process of achieving the goal of using cheap crude oil and that the highest degree of blending optimization requires continuous upgrading of the simulation model by adapting to real analytical trends.

Berna *et al* (2008) also studied the experimental and numerical simulation of soil recovery from oil Shales by electrical heating. The result also shows that the experimental and numerical results of the field-scale oil recovery from Shales by electrical heating could be technically and economically viable.

As in other crude oil producing nation, the hydrocarbon constituent play a significant role on the quantity of crude oil that is expected to produced during the production period. Hafner, Tagliapieta and Strasser (2018) on the role of hydrocarbon in African's energy mix. Al-jarri and Startzman (1997) also in a study analyzed the world crude oil production trends and reviews historical crude oil production of 67 countries. The study observed that a weak hydrocarbon constituent is associated with a relatively small increase in the production of crude oil whereas a strong hydrocarbon constituent is associated with a relatively larger increase in the quantity of crude oil that is being produced.

This two patterns of crude oil production due to the inclusion of a weak hydrocarbon constituent and a strong hydrocarbon constituent can go a long way to indicate region of profitability in terms of the revenue that is derived from the crude oil produced on the assumption of an annual number of barrels of crude oil that is produced that is linked to the total revenue derived, this ideas will translated into numerical simulated data of crude oil in the unit of barrels that is being produced from crude oil wells that is located at close proximity within the Bayelsa State environment of Nigeria.

Having systematically reviewed the academic literature on the modelling of crude oil production, we have found a popular application of the first order differential equation (ODE) with a fewer application of partial differential equations (PDE), differential equation, stochastic ordinary differential equations, delayed differential equation and functional differential equation to mention but a few. However, there is dealt of numerical simulated mathematical model that can be used to solve and analyze the problem of crude oil that is produced overtime especially in the context of two (2) interacting crude oil well with Lotka Volterra option.

Lotka Volterra equations, also known as the predator–Prey equations are a pair of first order non–linear differential equation, frequently used to describe the dynamics of biological system in which two species interact, one as a predator and the other as a prey. In other words, the Lotka Volterra equations are simple mathematical model of the population dynamics of two species competing for same common resources. In this context, it is two (2) crude oil wells competing for crude oil that is produced overtime.

It is against this background that this study have chosen the computational method of ordinary differential equation of order 45 (ODE45) numerical simulations.

2.1 Materials and Methods

2.2 Mathematical Formulations

This study have considered two systems of first order differential equation (ODE) under the simplifying assumptions as a guide to the mathematical formulations or equations that describe the production of crude oil over time. The rate of change of crude oil that is produced is directly proportional to the quantity of crude oil produced over time. And the rate of change of crude oil that is produced is inhibited by the intra competition coefficient. In the situation of two interacting oil wells, we assume that the inter competition coefficient can either play a positive role or a negative role on the quantity of crude oil that is produced. The initial quantity of crude oil that is produced at the initial time is specified to be positive. Also the duration of crude oil that is produced is assumed to be on a monthly basis which is considered with every thirty (30) days of crude oil production timing.

Following Ofor (2020), the study has considered the following dynamical system that describes the two interacting crude oil wells;

$$\frac{dc_1}{dt} = \alpha_1 c_1 - \beta_1 c_1^2 - r_1 c_1 c_2 \quad (1)$$

$$\frac{dc_2}{dt} = \alpha_2 c_2 - \beta_2 c_2^2 - r_2 c_1 c_2 \quad (2)$$

With the following initial conditions

$$C_1(0) = c_{10} > 0, \quad C_2(0) = c_{20} > 0$$

Where,

$C_1(t)$ and $C_2(t)$ defines the quantity of crude oil that is produced over time in the first oil well and the second oil well respectively, α_1 and α_2 defines the two growth rate parameter for the first oil well and the second oil well respectively, β_1 and β_2 defines the two intra-competition coefficient for the first oil well and the second oil well respectively and r_1 and r_2 defines the two inter-competition coefficient for the first oil well and the second oil well respectively

2.3 Mathematical Preliminary Steady State Solutions:

The focus of this section is to derive the steady state solutions

2.3.1 Steady State Solutions

Consider the following equations

$$\frac{dc_1}{dt} = \alpha_1 c_1 - \beta_1 c_1^2 - r_1 c_1 c_2$$

$$\frac{dc_2}{dt} = \alpha_2 c_2 - \beta_2 c_2^2 - r_2 c_1 c_2$$

Following Ekaka-a & Uka, (2012), Nwagor et al (2020) at a steady state solution all rate of change equal to zero, hence we shall find the values of c_1 and c_2 that satisfy

$$\frac{dc_1}{dt} = \frac{dc_2}{dt} = 0, \text{ If } \frac{dc_1}{dt} = 0$$

Then

$$\alpha_1 c_1 - \beta_1 c_1^2 - r_1 c_1 c_2 = 0$$

$$c_1 (\alpha_1 - \beta_1 c_1 - r_1 c_2)_1 = 0$$

Similarly if $\frac{dc_2}{dt} = 0$, Then

$$c_2 (\alpha_2 - \beta_2 c_2 - r_2 c_1)_2 = 0$$

Case I

$$\text{If } (\alpha_1 - \beta_1 c_1 - r_1 c_2)_1 \neq 0 \quad c_1 = 0$$

$$(\alpha_2 - \beta_2 c_2 - r_2 c_1)_2 \neq 0 \quad c_2 = 0$$

Therefore, the point $(c_1, c_2) = (0, 0)$ is steady state solution (sss).

Case 2

$$\text{If } c_1 \neq 0 \quad c_2 = 0, \quad \frac{dc_2}{dt} = 0$$

Case 3

$$\text{If } c_2 = 0 \quad c_1 = 0, \quad \frac{dc_1}{dt} = 0$$

Case 4

$$\text{If } c_1 \neq 0, \quad c_2 \neq 0$$

to calculate c_1 and c_2 using the giving assumed parameter values of;

$$r_1 = 0.025 \quad r_2 = 0.018, \beta_1 = 0.085, \quad \beta_2 = 0.075, \alpha_1 = 0.04, \alpha_2 = 0.12$$

$$c_1 = \frac{\alpha_1 \beta_1 - r_1 r_2}{\beta_1 \beta_2 - r_1 r_2}, \quad \alpha_1 \beta_1 = (0.04)(0.085) = 0.0034$$

$$r_1 \alpha_2 = (0.025)(0.12) = 0.003$$

$$\alpha_1 \beta_1 - r_1 \alpha_2 = 0.0034 - 0.003 = 0.0004$$

$$\beta_1 \beta_2 = (0.085)(0.075) = 0.06375$$

$$r_1 r_2 = (0.025)(0.018) = 0.00045$$

$$\beta_1 \beta_2 - r_1 r_2 = 0.06375 - 0.00045 = 0.0633$$

$$c_1 = \frac{\alpha_1 \beta_1 - r_1 \alpha_2}{\beta_1 \beta_2 - r_1 r_2} = \frac{0.0004}{0.0633} = 0.0063 > 0, \therefore c > 0$$

From c_1 ;

$$\alpha_1 \beta_1 = r_1 \alpha_2 > 0$$

$$\alpha_1 \beta_1 > r_1 \alpha_2$$

$$\therefore \alpha_1 > \frac{r_1 \alpha_2}{\beta_1}$$

$$\text{Hence } c_1 = \frac{\alpha_1 \beta_2 - r_1 \alpha_2}{\beta_1 \beta_2 - r_1 r_2} \text{ exist}$$

Provided

$$1. \quad \alpha_1 \beta_2 > r_1 \alpha_2$$

$$2. \quad \beta_1 \beta_2 > r_1 \alpha_2$$

Also

$$c_2 = \frac{\alpha_2 \beta_1 - r_1 r_2}{\beta_1 \beta_2 - r_1 r_2}$$

$$\alpha_2 \beta_1 = (0.12)(0.085) = 0.0102$$

$$\alpha_1 r_2 = (0.04)(0.018) = 0.00072$$

$$\begin{aligned}\alpha_2 \beta_1 - \alpha_1 r_2 &= 0,0102 - 0,00072 = 0,00948 \\ \beta_1 \beta_2 &= (0,085) (0,075) = 0,006375 \\ r_1 r_2 &= (0,025) (0,018) = 0,00045 \\ \beta_1 \beta_2 - r_1 r_2 &= 0,006375 - 0,00045 = 0,005925 \\ c_2 &= \frac{\alpha_2 \beta_1 - \alpha_1 r_2}{\beta_1 \beta_2 - r_1 r_2} = \frac{0,00948}{0,005925} \\ &= 1,6 > 0 \\ \therefore c_2 &= 1,6 > p\end{aligned}$$

Therefore points c_1 and c_2 ($c_1 c_2$) = (0.0675, 1.6)

Again from c_2

$$\alpha_2 \beta_1 - \alpha_1 r_2 > 0 \Rightarrow \alpha_2 \beta_1 > \alpha_1 r_2, \alpha_2 > \frac{\alpha_1 r_2}{\beta_1}$$

$$c_2 = \frac{\alpha_1 \beta_1 - \alpha_1 r_2}{\beta_1 \beta_2 - r_1 r_2} \text{ exist}$$

Provided

0. $\alpha_1 \beta_2 > \alpha_1 r_2$
1. $\beta_1 \beta_2 > r_1 r_2$

That is to say c_1 and c_2 exist provided

1. $\alpha_2 \beta_1 > \alpha_1 r_2$
2. $\beta_1 \beta_2 > r_1 r_2$

Therefore we obtained the following steady state solutions namely

1. The point (c_1 and c_2) = (0,0)
2. The point (c_1 and c_2) = (0.0675, 1.6)

3. Method of data analysis

In this study, we have considered the following step by step numerical methods.

Step 1: We have coded in matlab programming the two non-linear equations that are specified above when all the parameter values are fixed. This matlab solved program is controlled program.

Step 2: We have coded the same system of equations in which a chosen model parameter value is varied in order to study the effect of this variation on the crude oil that is produced in the two crude oil wells over time. This program is called uncontrolled matlab solved program.

Step 3: Both the controlled and the uncontrolled under twine matlab solution program are simulated to quantify the effect of varying a model parameter value on the quantifying of crude oil that is produced in the two connecting crude oil well.

Step 4: Examine the impact of a model parameter value when decreased, does this variation provide either a positive impact or a negative impact.

For example if a model parameter value is decreased and it produces depletion on the quantity of crude oil that is produced, then this will affect the revenue that is derived from crude oil that is produced. On the other hand, if a decrease variation of a model parameter value leads to a positive impact on the crude oil that is produced; this would relatively lead to a finite increase in the expected crude oil derived revenue which will have an impact on the geo-political regional sustainable development scenario.

For the purpose of clarity, we have utilized the following expression to calculate the expected proportion increase (EPI) in percentage terms due to the inclusion of a 0.08 environmental perturbation.

This formula is defined as follows:

- i. For the first crude oil well, $(EPI)_1\%$ can be calculated as follows:
 $(EPI)_1\%$ equal to c_1 (new) minus c_1 (old) divided by $c_1(\text{old})_1$ provided that c_1 (new) is bigger than c_1 (old) and c_1 (old) is not equal to zero.
 Expressing it mathematically, we have $(EPI)_1\% = \frac{C_1(\text{new}) - C_1(\text{old})}{C_1(\text{old})} \times 100$ provided $C_1(\text{new}) > C_1(\text{old}), C_1(\text{old}) \neq 0$
- ii. For the second crude oil well
 $(EPI)_2\% = \frac{C_2(\text{new}) - C_2(\text{old})}{C_2(\text{old})} \times 100$ provided $C_2(\text{new}) > C_2(\text{old})$
 $C_2(\text{old}) \neq 0$, where $C_1(\text{old}), C_1(\text{new}), C_2(\text{old}), C_2(\text{new})$, are solution trajectories or solution map data which are time dependent.

4. Results and Discussions

Table 1; Quantifying the effect of a random environmental perturbation value of 0.08 on the quantity of crude oil produced in million barrels on the context of two competing crude oil wells, using ODE 45 Numerical Simulation.

Scenario 1

LOP in days	$C_1(\text{old})$	$C_1(\text{new})$	$(EPI)_1\%$	$C_2(\text{old})$	$C_2(\text{new})$	$(EPI)_2\%$
1	5000	0.5000	0.0000	0.5000	0.5000	0.00
31	1.3539	3.6617	170.46	1.1523	3.1099	169.89
61	2.7879	9.0259	223.75	2.1757	7.875	261.95
91	4.0934	10.5786	158.43	3.2007	9.0643	183.20
121	4.7659	10.6223	122.88	3.8561	9.6716	150.81
151	5.0136	10.2341	104.13	4.1633	9.3712	125.09
181	5.0938	10.1431	99.13	4.2868	9.6379	124.83
211	5.1176	10.4222	103.65	4.3326	9.0797	109.57
241	5.1247	10.2901	100.79	4.3492	9.426	116.73
271	5.1275	10.4629	104.05	4.3559	9.446	116.86
301	5.1276	9.9439	93.93	4.3574	9.5615	119.43
331	5.1284	10.0474	95.92	4.3588	9.4215	116.15
361	5.128	10.156	98.05	4.3587	9.4629	117.10
391	5.1282	9.8918	92.89	4.3588	9.4874	117.66

421	5.1284	10.0307	95.59	4.359	9.7606	123.92
451	5.1279	10.0598	96.18	4.3588	9.4223	116.17
481	5.1283	10.0514	96.00	4.359	9.5933	120.08
511	5.1282	10.0497	95.97	4.359	9.328	113.99
541	5.128	10.4528	103.84	4.3589	9.7343	123.32
571	5.1284	10.1302	97.53	4.359	9.3909	115.44
601	5.1281	10.428	103.35	4.359	9.666	121.75
631	5.1282	10.5122	104.99	4.359	9.362	114.77
661	5.1283	10.1505	97.93	4.359	9.5173	118.34
691	5.1281	10.1909	98.73	4.359	9.33	114.04
721	5.1283	10.3159	101.16	4.359	9.2976	113.30
751	5.1282	10.5804	106.32	4.359	9.507	118.10

Table 2; Quantifying the effect of a random environmental perturbation value of 0.08 on the quantity of crude oil produced in million barrels on the context of two competing crude oil wells, using ODE 45 Numerical Simulation.

Scenario 2

LOP in days	C ₁ (old)	C ₁ (new)	(EPI) ₁ %	C ₂ (old)	C ₂ (new)	(EPI) ₂ %
1	0.5	0.5	0.00	0.5	0.5	0.00
31	1.3539	3.4691	156.23	1.1523	3.2315	180.44
61	2.7879	8.8834	218.64	2.1757	7.9236	264.19
91	4.0934	10.1815	148.73	3.2007	9.0993	184.29
121	4.7659	10.041	110.68	3.8561	9.2646	140.26
151	5.0136	10.017	99.80	4.1633	9.9098	138.03
181	5.0938	10.1684	99.62	4.2868	10.0145	133.61
211	5.1176	10.1578	98.49	4.3326	9.4849	118.92
241	5.1247	10.5585	106.03	4.3492	9.7595	124.40
271	5.1275	10.3236	101.34	4.3559	9.62	120.85
301	5.1276	10.6877	108.43	4.3574	9.3779	115.22
331	5.1284	10.3033	100.91	4.3588	9.5604	119.34
361	5.128	10.0396	95.78	4.3587	9.1736	110.47
391	5.1282	10.357	101.96	4.3588	9.4101	115.89
421	5.1284	10.1701	98.31	4.359	9.5346	118.73
451	5.1279	9.8636	92.35	4.3588	9.3948	115.54
481	5.1283	10.3248	101.33	4.359	9.6916	122.34
511	5.1282	10.425	103.29	4.359	9.6098	120.46
541	5.128	9.8886	92.84	4.3589	9.8481	125.93
571	5.1284	9.839	91.85	4.359	9.6315	120.96

601	5.1281	9.8672	92.41	4.359	9.3407	114.29
631	5.1282	9.9932	94.87	4.359	9.2433	112.05
661	5.1283	10.6027	106.75	4.359	9.2859	113.03
691	5.1281	10.1885	98.68	4.359	9.4427	116.63
721	5.1283	10.2544	99.96	4.359	9.1482	109.87
751	5.1282	10.5634	105.99	4.359	9.4203	116.11

Table3: Quantifying the effect of a random environmental perturbation value of 0.08 on the quantity of crude oil produced in million barrels on the context of two competing crude oil wells, using ODE 45 Numerical Simulation.

Scenario 3

LOP in days	C ₁ (old)	C ₁ (new)	(EPI) ₁ %	C ₂ (old)	C ₂ (new)	(EPI) ₂ %
1	0.5	0.5	0.00	0.5	0.5	0.00
31	1.3539	3.6237	167.65	1.1523	3.0586	165.43
61	2.7879	8.9098	219.59	2.1757	7.6643	252.27
91	4.0934	9.9558	143.22	3.2007	8.8874	177.67
121	4.7659	10.5431	121.22	3.8561	9.6046	149.08
151	5.0136	10.2494	104.43	4.1633	9.3093	123.60
181	5.0938	10.3295	102.79	4.2868	9.5557	122.91
211	5.1176	10.0006	95.42	4.3326	9.5988	121.55
241	5.1247	9.9423	94.01	4.3492	9.7197	123.48
271	5.1275	10.24	99.71	4.3559	9.9095	127.50
301	5.1276	10.2657	100.20	4.3574	9.3733	115.11
331	5.1284	10.2787	100.43	4.3588	9.8287	125.49
361	5.128	10.5625	105.98	4.3587	9.6799	122.08
391	5.1282	10.594	106.58	4.3588	9.6451	121.28
421	5.1284	10.4181	103.15	4.359	9.6136	120.55
451	5.1279	10.0217	95.43	4.3588	9.5374	118.81
481	5.1283	10.1518	97.96	4.359	9.3591	114.71
511	5.1282	10.3375	101.58	4.359	9.2901	113.12
541	5.128	10.4825	104.42	4.3589	9.368	114.92
571	5.1284	10.1203	97.34	4.359	9.7879	124.54
601	5.1281	10.0784	96.53	4.359	9.2353	111.87
631	5.1282	10.0549	96.07	4.359	9.4935	117.79
661	5.1283	10.3211	101.26	4.359	9.5566	119.24
691	5.1281	10.2482	99.84	4.359	9.2812	112.92
721	5.1283	10.4598	103.96	4.359	9.482	117.53
751	5.1282	10.2257	99.40	4.359	9.3623	114.78

Table.4: Quantifying the effect of a random environmental perturbation value of 0.08 on the quantity of crude oil produced in million barrels on the context of two competing crude oil wells, using ODE 45 Numerical Simulation.

Scenario 4

LOP in days	C ₁ (old)	C ₁ (new)	(EPI) ₁ %	C ₂ (old)	C ₂ (new)	(EPI) ₂ %
1	0.5	0.5	0.00	0.5	0.5	0.00
31	1.3539	3.6617	170.46	1.1523	3.1099	169.89
61	2.7879	9.0259	223.75	2.1757	7.875	261.95
91	4.0934	10.5786	158.43	3.2007	9.0643	183.20
121	4.7659	10.6223	122.88	3.8561	9.6716	150.81
151	5.0136	10.2341	104.13	4.1633	9.3712	125.09
181	5.0938	10.1431	99.13	4.2868	9.6379	124.83
211	5.1176	10.4222	103.65	4.3326	9.0797	109.57
241	5.1247	10.2901	100.79	4.3492	9.426	116.73
271	5.1275	10.4629	104.05	4.3559	9.446	116.86
301	5.1276	9.9439	93.93	4.3574	9.5615	119.43
331	5.1284	10.0474	95.92	4.3588	9.4215	116.15
361	5.128	10.156	98.05	4.3587	9.4629	117.10
391	5.1282	9.8918	92.89	4.3588	9.4874	117.66
421	5.1284	10.0307	95.59	4.359	9.7606	123.92
451	5.1279	10.0598	96.18	4.3588	9.4223	116.17
481	5.1283	10.0514	96.00	4.359	9.5933	120.08
511	5.1282	10.0497	95.97	4.359	9.328	113.99
541	5.128	10.4528	103.84	4.3589	9.7343	123.32
571	5.1284	10.1302	97.53	4.359	9.3909	115.44
601	5.1281	10.428	103.35	4.359	9.666	121.75
631	5.1282	10.5122	104.99	4.359	9.362	114.77
661	5.1283	10.1505	97.93	4.359	9.5173	118.34
691	5.1281	10.1909	98.73	4.359	9.33	114.04
721	5.1283	10.3159	101.16	4.359	9.2976	113.30
751	5.1282	10.5804	106.32	4.359	9.507	118.10

Table 5: Quantifying the effect of a random environmental perturbation value of 0.08 on the quantity of crude oil produced in million barrels on the context of two competing crude oil wells, using ODE 45 Numerical Simulation.

Scenario 5

LOP in days	C ₁ (old)	C ₁ (new)	(EPI) ₁ %	C ₂ (old)	C ₂ (new)	(EPI) ₂ %
1	0.5	0.5	0.00	0.5	0.5	0.00
31	1.3539	3.7428	.176.45	1.1523	3.1286	171.51
61	2.7879	9.0811	225.73	2.1757	7.506	244.99
91	4.0934	10.1069	146.91	3.2007	9.5618	198.74
121	4.7659	10.0655	111.20	3.8561	9.5937	148.79
151	5.0136	10.2976	105.39	4.1633	9.6837	132.60
181	5.0938	10.4775	105.69	4.2868	9.461	120.70
211	5.1176	10.486	104.90	4.3326	9.5785	121.08
241	5.1247	10.8893	112.49	4.3492	9.5708	120.06
271	5.1275	10.3027	100.93	4.3559	9.7463	123.75
301	5.1276	10.0425	95.85	4.3574	9.7161	122.98
331	5.1284	9.8088	91.26	4.3588	9.3297	114.04
361	5.128	10.268	100.23	4.3587	9.9246	127.70
391	5.1282	10.4788	104.34	4.3588	9.3276	113.99
421	5.1284	10.2385	99.64	4.359	9.6564	121.53
451	5.1279	10.4326	103.45	4.3588	9.6128	120.54
481	5.1283	10.3358	101.54	4.359	9.6879	122.25
511	5.1282	10.2753	100.37	4.359	9.3892	115.40
541	5.128	10.1194	97.34	4.3589	9.6382	121.12
571	5.1284	10.2661	100.18	4.359	9.7635	123.98
601	5.1281	10.4976	104.71	4.359	9.4831	117.55
631	5.1282	10.1745	98.40	4.359	9.4557	116.92
661	5.1283	10.4277	103.34	4.359	9.5033	118.02
691	5.1281	10.0647	96.27	4.359	9.5346	118.73
721	5.1283	10.2638	100.14	4.359	9.5587	119.29
751	5.1282	10.4276	103.34	4.359	9.6978	122.48

Table 6: Quantifying the effect of a random environmental perturbation value of 0.08 on the quantity of crude oil produced in million barrels on the context of two competing crude oil wells, using ODE 45 Numerical Simulation.

Scenario 6

LOP in days	$C_1(\text{old})$	$C_1(\text{new})$	$(EPI)_1\%$	$C_2(\text{old})$	$C_2(\text{new})$	$(EPI)_2\%$
1	0.5	0.5	0.00	0.5	0.5	0.00
31	1.3539	3.6964	173.02	1.1523	3.1969	177.44
61	2.7879	8.7456	213.70	2.1757	7.9467	265.25
91	4.0934	10.2298	149.91	3.2007	9.2793	189.91
121	4.7659	9.9124	107.99	3.8561	9.4337	144.64
151	5.0136	10.4149	107.73	4.1633	9.6259	131.21
181	5.0938	10.4896	105.93	4.2868	9.892	130.75
211	5.1176	10.5595	106.34	4.3326	9.2266	112.96
241	5.1247	9.8416	92.04	4.3492	9.6088	120.93
271	5.1275	10.0444	95.89	4.3559	9.5996	120.38
301	5.1276	9.9379	93.81	4.3574	9.2016	111.17
331	5.1284	10.5643	106.00	4.3588	9.275	112.79
361	5.128	10.4328	103.45	4.3587	9.8981	127.09
391	5.1282	10.2798	100.46	4.3588	9.7515	123.72
421	5.1284	9.801	91.11	4.359	9.4828	117.55
451	5.1279	10.078	96.53	4.3588	9.9421	128.09
481	5.1283	10.5112	104.96	4.359	9.5616	119.35
511	5.1282	10.3515	101.85	4.359	9.1743	110.47
541	5.128	10.2573	100.03	4.3589	9.7058	122.67
571	5.1284	10.5585	105.88	4.359	9.5338	118.72
601	5.1281	10.2417	99.72	4.359	9.7875	124.54
631	5.1282	10.1313	97.56	4.359	9.3848	115.30
661	5.1283	10.3874	102.55	4.359	9.0828	108.37
691	5.1281	10.7191	109.03	4.359	9.394	115.51
721	5.1283	10.3679	102.17	4.359	9.6484	121.34
751	5.1282	10.3336	101.51	4.359	9.5954	120.13

In the context of two competing crude oil wells, the study have quantified the effects of a random environmental perturbation value of 0.08 on the quantity of crude oil that is produced in Bayelsa oil field for every thirty days experimental time using MATLAB ODE 45 numerical scheme.

Look at the scenario 1 (one) of this section, the numerically simulated results as displayed in Table1, as the length of production in the unit of days changes for every thirty (30) days, we have also observed the quantity of crude oil that is produced in the unit of millions of barrels in the crude oil well denoted by C_1 (old) and C_2 (old) grow from initial condition value of 0.05 at

day one (1) to 5.1282 and 4.3590 respectively at day 751 in the absent of a random environmental perturbation, but the inclusion of a 0.08 random environmental perturbation has shown some elements of a mild fluctuation in the C_1 (new) and C_2 (new) data which dominantly out weigh in magnitude the C_1 (old) data and C_2 (old) data which indicate that the quantity of crude oil that is produced due to the inclusion of a 0.08 random environmental perturbation has increase the quantity of crude oil that is produced in the two competing crude oil wells in which in the C_1 (new) data, it is predicted to 10.5804 which is bigger than 5.1282 at day 751 producing an estimated proportion increase $(EPI)_1$ value of 105.32.

Similarly, the C_2 (new) data at day 751 has been predicted to be 9.5070 which is also bigger than the C_1 (old) value of 4.3590, producing an estimated proportion increase value of 118.10.

Following scenario 2 (two) of Table 4.2.2, we have obtained the following numerical simulation results at 751. Irrespective of scenario 1 and scenario 2 predicted value of crude oil that is produced in the two competing crude oil wells, the converging values of $(C_1$ (old) data and C_2 (old) data remain at the same value of 5.1282 and 4.3590 respectively whereas the C_1 (new) value converges to 10.5634 which has produced an estimated proportion increase value 105.99 while the C_2 (new) data has produced the converging value of 9.4203 which has similarly produced an estimated proportion increase value of 116.11.

Between scenario 1 and scenario 2 data of Table 1 and Table 2, the second crude oil well generally benefit relatively from the increase in the quantity of crude oil that is produced than the first crude oil well in the context of the two (2) competing crude oil wells.

From scenario 3 (three) of Table 3, we also obtained the same numerical simulation results at day 751 for C_1 (old) and C_2 (old) data respectively. This is to say their converging values remain the same with of scenario 1 of Table 4.2.1 and scenario 2 of Table 4.2.2, whereas the C_1 (new) data in Table 4.2.3 converges to 10.2257 which produced an estimated proportion increase of 99.40 while the C_2 (new) data with a converging value of 9.3623 produces an estimated proportion increase of 114.78.

Again in scenario 4 (four) of Table 4, the same observation still holds in C_1 (old) data and C_2 (old) data respectively. But the inclusion of a random environmental perturbation value of 0.08 produces different values of C_1 (new) and C_2 (new) respectively. The C_1 (new) data converges to 10.5804 producing an estimated proportion increase value of 106.32 whereas the C_2 (new) data converges to 9.5070 also producing an estimated proportion increase value of 118.10.

Furthermore, in scenario 5 (five) of Table 5 at day 751, the C_1 (new) data converges to 10.4276 which produced an estimated proportion increase value of 103.34 while the C_2 (new) data converges to 9.6978 also produced an estimated proportion increase value of 122.48.

This trend also continues in scenario six (6) of Table 6 to scenario twenty (20). From scenario 1 to scenario 20 (twenty), we observed that the second crude oil well generally benefits relatively from the increase in the quantity of crude oil that is produced than the first crude oil well. Also observed is the rise and fall in the quantity of crude oil that is produced in the two competing crude oil wells as the trend continues. This observation agrees with the study of Paul, Isaiah and Archana (2018).

5. Conclusion

In the context of ordinary differential equation (ODE) simulation modelling of crude oil production, this study which include the effect of a random environmental perturbation on the quantity of crude oil production for two (2) competing crude oil well in the Bayelsa State of

Nigeria. This study has successfully applied the method of numerical simulation to predict the quantities of crude oil that is produced on two competing crude oil wells that are allocated in the oil rich Bayelsa State of Nigeria. The key results of this study have predicted that the effect of random environmental perturbation value of 0.08 which is four (4) times larger than the first random perturbation value generally leads to a larger volume of crude oil that is produced as shown in Tables 1 to 6.

For the purpose of policy, the implication of this results is of the fact that a weak hydrocarbon constituent which drives the definition of the random environmental perturbation leads to a smaller volume of the crude oil that is being produced in the two competing crude oil wells whereas the same hydrocarbon constituent with a higher grade has the capacity of producing a relatively bigger volume of crude oil that is produced which has the enhance to attract better revenue derived from crude oil production.

On the basis of this insight, the study will recommend that the federal government of Nigeria should adopt alternative approach that will enhance productivity of higher value crude for optimal profit and hence consider a high grade hydrocarbon constituent that will enable high income generation and the sustainability of our national economic values. This compliments the earlier contribution on the prediction of Nigerians oil depletion that was based on Hubbert's model for renewable energy and the study on an aspect of the numerical simulation on heavy oil production that uses the method of silo combustion to form a new frontier of knowledge which has presently looked at the implication of using a stronger hydrocarbon constituent on the quantity of crude oil that is expected to be produced in the creeks of Bayelsa State in Nigeria.

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